

Atmospheric Extinction by Dust Particles as Determined from Three-Color Photometry of the Lunar Eclipse of 19 December 1964*

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Three-color photometric observations of the eclipsed moon were carried out with a 12-in. reflector during the total eclipse on 19 December 1964, with the purpose of obtaining data for determining the dust content in the earth's atmosphere. In order to make possible a direct comparison with similar data obtained with the same equipment during the previous eclipse (30 December 1963), the photoelectric measurements were centered principally on the Mare Crisium region, except for a part of the observations which were made near Mare Frigoris. The maximum decreases in brightness at midtotality, which amounted to 12.6, 13.8, and 15.9 mag. in V , B , and U colors, respectively, were much less than those obtained at the previous eclipse but were still considerably larger than those of a normal eclipse. As opposed to the December 1963 eclipse in which no reddening effect was observed, the maximum increases of the $B-V$ and $U-B$ color indices were found to be 1.1 and 2.2 mag., respectively.

Theoretical calculations of the intensity distribution in the umbral region are made for various models of the earth's atmosphere with different contents of dust and water particles. Comparisons with the observations indicate that the amount of dust and water particles at the time of the eclipse was such that it produced an additional extinction of 0.04 mag. per air mass. The corresponding value determined for the December 1963 eclipse was about 0.14 mag. per air mass.

I. INTRODUCTION

AS is well known, the intensity and color distribution of sunlight in the earth's shadow, observable at the time of a total eclipse of the moon, provide useful data for studying the structure of the upper atmosphere, especially the location and amount of various particles which cause attenuation of light. Hansen and Matsushima (1966) have calculated the distributions of the umbral light for various models of the earth's atmosphere, and, using the three-color photoelectric observation of the 30 December 1963 eclipse reported in the previous paper (Matsushima and Zink 1964), they concluded that the unusually dark and nonreddened eclipse of the moon could be explained by assuming an abnormally large extinction presumably caused by dust spread by the eruption of Mt. Agung on 17 March 1963. They also showed that accurate light curves near the umbra limb are most sensitive in determining the atmospheric extinction due to dust and other particles. If the cause of the unusually dark eclipse was indeed due to an abnormal amount of atmospheric dust of volcanic origin, such an additional dust layer would be likely to remain in part for a few years before the extinction returned to the normal amount. The next total eclipse visible in Iowa appeared to be an excellent opportunity to examine this point, and the three-color photometric observation was repeated on 19 December at the old Country Observatory of the University of Iowa. (This observatory site

was closed after the completion of the new Observatory in the spring 1965.) To facilitate a comparison with the data of the earlier eclipse, the observation was centered primarily on Mare Crisium and the same set of instruments was used. The sky condition was not as good as it was during the previous eclipse, the region near the moon being partly covered by thin clouds, but successful measurements were made without interruption during the first half of the total phase of the eclipse. In the present paper, we report the result of the observations together with theoretical interpretations of the data to determine the atmospheric dust content.

II. OBSERVATION

The observations were made with a 12-in. Cassegrain reflector of slightly reduced aperture and conventional three-color photometric equipment. This equipment had already been used satisfactorily for observations of the December 1963 eclipse and was employed again for the present observations without modification. A more detailed description of the equipment and the observing procedure was given in the previous paper (Matsushima and Zink 1964). Briefly, the procedure is to measure intensities of the selected lunar area and of standard stars in rapid sequence through the visual, blue, and ultraviolet filters. After reference to a radium standard, the sequence is reversed. The entire sequence is then repeated for sky background and occasionally a check is made on the amount of dark current. The angular diameter of the observed area is determined by a series of diaphragms and must be changed during the course of the eclipse in order to keep the output current adjusted within the range of the recorder and integrator. The smallest of these diaphragms provides a field of about 6".7 angular diameter for observations of the uneclipsed moon. The largest diaphragm giving

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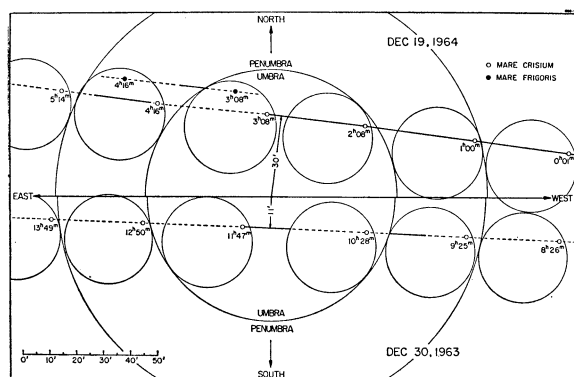


FIG. 1. Geometric relation between the paths of the moon during the eclipses of 19 December 1964 and 30 December 1963. The time of each contact is given in Universal Time.

a field of about $150''$ angular diameter is necessary for ultraviolet readings well within the umbra.

The observations involved two areas of the lunar surface. The central portion of Mare Crisium was observed during its entry into the umbra, since, as mentioned in the preceding section, it provided a reliable comparison for the different sets of eclipse data. A second area, Mare Frigoris, was also observed since the bright sky background and poor lunar contrast during the umbral stage of the eclipse made difficult an accurate positioning of the Mare Crisium image on the diaphragm of the photometer. This period was due to the presence of a nearby bright feature, readily visible even within the umbra, which incidentally served as a useful landmark in repositioning the telescope after it had been moved off the moon for sky background readings.

The paths of the moon during the 1963 and 1964 eclipses are compared in Fig. 1. The positions of the two observed areas at each contact are shown by an open (Mare Crisium) and a solid (Mare Frigoris) circle, and the time indicated gives the Universal Time of each contact. The observed points are connected by broken lines which indicate the paths of these areas during the course of the eclipses, and the solid portions of these lines represent the periods during which actual measurements were continuously taken. The conversion from the observed time to the angular distance from the umbra center can be made graphically with sufficient accuracy on a large scale of such a diagram. We see in Fig. 1 that if a spherical symmetry is assumed for the distribution of light in the shadow, the data for the two eclipse observations can be combined for comparing the radial distributions of the shadow brightness.

Observing conditions were much less favorable than those prevailing at the time of the December 1963 eclipse. During the observations, a light haze and thin wispy clouds produced a considerable increase in the sky brightness in the vicinity of the moon. These atmospheric conditions resulted in significant changes

in sky transparency from moment to moment and made impossible the determination and use of precise extinction coefficients. It was necessary to adopt extinction coefficients which seemed to best represent average values for the period over which the lunar observations were made. A necessary consequence was a larger than desired scatter in the points determining the final light curves. The values adopted for average extinction coefficients were 0.30, 0.44, and 0.75 mag. per unit air mass for the V , B , and U colors, respectively.

In the reduction of the data, an independent magnitude scale was established for each of the three colors. The magnitudes of the observed areas just prior to the eclipse were arbitrarily made zero on each scale. These magnitude scales may be interpreted in terms of true magnitude scales for a specific area of the lunar surface through comparisons with standard star observations. For the purpose of this paper, however, it is not necessary that the scales be adjusted to represent actual magnitudes. The arbitrary blue magnitude at any instant minus the corresponding arbitrary visual magnitude represents the increase in the $B-V$ color index over the uneclipsed value. The color indices of the uneclipsed Mare Crisium were obtained through comparisons with standard stars. In order to minimize color effects introduced by the haze, the greatest stress was placed on the comparisons with the star η Peg which had photoelectric color indices ($B-V$, +0.84; $U-B$, +0.51) more nearly resembling those of the lunar areas than other conveniently located standard stars in the appropriate magnitude range.

III. RESULT OF OBSERVATION

The final results for the brightness in each color normalized through the procedure discussed in the previous section are summarized in Table I. The

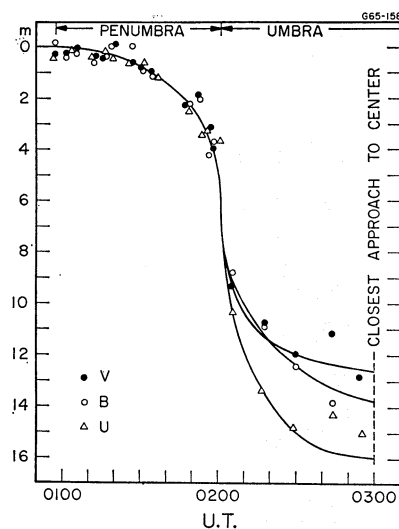


FIG. 2. Magnitude-color variation of Mare Crisium before the midtotality of the 19 December 1964 eclipse.

TABLE I. Three-color magnitudes of Mare Crisium and Mare Frigoris during the total eclipse of 19 December 1964.

UT	V r	m_V	UT	B r	m_B	UT	U r	m_U
Mare Crisium								
01 ^h 00 ^m 55 ^s	−30'.4	^m 21	01 ^h 01 ^m 15 ^s	−30'.2	^m 32	01 ^h 01 ^m 45 ^s	−30'.0	^m 27
01 05 25	−28.0	.04	01 05 00	−28.2	.24	01 02 45	−29.5	.26
01 12 20	−24.4	.32	01 11 50	−24.7	.60	01 11 05	−25.2	.42
01 15 20	−22.8	.42	01 15 45	−22.6	.39	01 16 15	−22.3	.29
01 19 55	−20.1	−.10	01 19 35	−20.4	−.06	01 19 05	−20.6	.49
01 26 50	−16.7	.60	01 26 25	−17.0	−.05	01 25 35	−17.3	.70
01 29 45	−15.0	.81	01 30 25	−14.8	.88	01 31 00	−14.4	.68
01 33 55	−12.9	.98	01 34 25	−12.7	1.11	01 34 50	−12.4	1.22
01 47 20	−6.0	2.27	01 47 50	−5.8	2.23	01 48 20	−5.5	2.54
01 52 15	−3.8	1.87	01 52 55	−3.4	2.03	01 53 25	−3.3	3.56
01 56 45	−1.7	3.12	01 56 25	−1.8	4.21	01 56 05	−2.1	3.28
01 57 45	−1.2	3.95	01 58 20	−.9	3.69	01 58 50	−.6	3.69
02 04 55	2.2	9.37	02 05 20	2.4	8.79	02 05 45	2.6	10.38
02 18 15	7.8	10.72	02 17 55	7.7	10.88	02 17 05	7.3	13.47
02 30 15	11.9	12.01	02 29 45	11.4	12.41	02 29 10	11.5	14.83
02 43 40	15.2	11.18	02 44 05	15.3	13.82	02 44 30	15.4	14.40
02 54 00	16.5	12.82				02 55 00	16.6	15.03
Mare Frigoris								
03 23 45	1.0	10.32						
03 26 50	−.1	4.96	03 26 30	.0	5.25	03 25 55	.2	6.43
03 28 40	−.8	4.39	03 29 00	−.8	5.40	03 29 20	−1.0	5.17
03 31 25	−1.8	3.83	03 31 00	−1.8	3.40	03 30 35	−1.5	3.75
03 32 40	−2.4	3.29	03 33 00	−2.5	3.75	03 33 30	−2.7	3.53
03 37 30	−4.4	2.66	03 37 55	−4.5	2.48	03 38 25	−4.7	2.75
03 40 35	−5.6	2.20	03 40 10	−5.4	2.21	03 39 45	−5.3	2.36
Mare Crisium								
04 24 05	−11.6	1.22	04 25 20	−12.2	1.14			
04 25 45	−12.0	1.30				04 25 50	−12.5	1.23
04 28 20	−13.9	.19	04 28 05	−13.7	.88	04 27 40	−13.5	1.00
04 49 45	−25.5	.41	04 49 15	−25.2	.24	04 48 30	−24.8	.35
04 52 55	−27.3	.19	04 53 15	−27.4	−.46	04 53 40	−27.7	.07

values, designated as r , in the second column of each color give the instantaneous position of the observed point with respect to the umbra in terms of the radial distance in minutes of arc from the umbra limb, increasing positively towards the center of the umbra. Since the angular radius of the umbra was about 45' at the time of this eclipse, the distance from the umbra center may be obtained by subtracting from this value the values in the second column. The observed brightness in magnitudes is given in the third column of each section with the brightness of the corresponding point outside of the eclipse being taken as 0 mag. The observation of Mare Crisium was continued without interruption until it reached the point nearest the umbra center at the distance of about 30'. Because of the observational difficulties mentioned in the previous section, the observed area was shifted to the Mare Frigoris region for about 20 min and then returned to Mare Crisium again after it entered the penumbral region.

The color variations with respect to time are shown in Fig. 2, in which the magnitudes of Mare Crisium before it reached the midtortality are plotted against the times of observation in Universal Time. The three curves drawn through the observed points are not the empirically smoothed curves as in the usual case but

represent the theoretical curves computed for the adopted model of the earth's atmosphere which are discussed in the following section. However, except for the last two points in U , the curves appear to fit the observed points well. As Mare Crisium approached nearest the umbra center, the other side of the moon began to leave the umbra, and the scattered sky light probably affected the ultraviolet measurements by the largest amount.

A reasonable estimate of the random error introduced by the rapid variations in sky transparency may be obtained from the observations of Mare Crisium just prior to the eclipse. The single measurement probable errors in the V , B , and U values were found to be ± 0.182 , ± 0.151 and ± 0.309 mag., respectively. These observations, however, were made through a relatively large air mass (of the order of 2.6) and some reduction in these errors should be expected for the later eclipse observations due to the smaller air mass involved.

The probable errors in the mean V , B , and U mag. determined for the uneclipsed Mare Crisium area were ± 0.069 , ± 0.057 , and ± 0.117 mag., respectively. These errors are comparable to those of the same region determined prior to the December 1963 eclipse. The December 1964 magnitudes were compared to the 1963 magnitudes and showed satisfactory agreement within

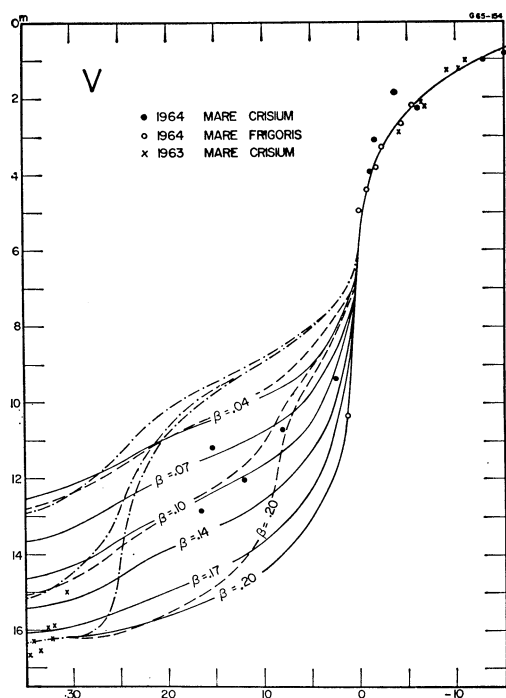


FIG. 3. Comparison of the observed V magnitude and the theoretical calculations for various models. Three different upper limits for the dust layer, H , assumed in the calculation are distinguished by solid (—, $H=25$ km), dashed (---, $H=20$ km), and combined dashed and dotted lines (— · — · —, $H=10$ km). β denotes an assumed vertical extinction coefficient in V .

the probable errors involved. The probable errors in the individual measurements during the progress of the eclipse cannot be determined, but it is unlikely that they exceeded those stated above for the uneclipsed moon observations until the observed areas were dimmed by about 10 mag. For all observations in which the brightness of the area was decreased by more than 10 mag., the random variations in sky brightness became the most serious source of error. The single-measurement probable errors are not easily determined since there was a steady decrease in sky background as the brightness of the observed area decreased. It is estimated, however, that a single-measurement probable error of ± 0.7 mag. for all three colors is appropriate for determinations of the magnitudes well within the umbra.

From Fig. 2, we estimate that the maximum decreases of brightness in V , B , and U are 12.6, 13.8, and 15.9 mag., respectively, at a radial distance of 30' from the center of the umbra. Unfortunately the magnitudes at the corresponding distance for the previous eclipse were not obtained (Fig. 1). However, the maximum decrease during that eclipse in V was 16.25 mag. at a distance of about 11' from the umbra center (Matsushima and Zink 1964). As will be shown in the following section, the best fitting theoretical curve for V color chosen in Fig. 2 yields about 13.6 mag. at the corresponding distance from the center of the umbra (see

the solid curve for $\beta=0.10$ in Fig. 3), and there seems to be no doubt that the moon was considerably brighter and redder than at the time of the previous eclipse.

As discussed in the preceding section, the vertical difference between two curves in Fig. 2 gives the change in the color indices at each instant as compared to the uneclipsed values. The $B-V$ and $U-B$ color indices for the uneclipsed Mare Crisium were found to be $+0.81 \pm 0.090$ and $+0.43 \pm 0.130$ which are only slightly higher than the corresponding color indices $+0.79 \pm 0.077$ and $+0.41 \pm 0.088$ found on the night of the December 1963 eclipse.

IV. THEORETICAL ANALYSIS

The principal causes of changes in the light intensity as it passes through the atmosphere are refraction which simply spreads the light over a wider area and extinction which actually removes a fraction of the light from the incident beam. In extinction are included both the scattering and absorption of light by various particles in the atmosphere. Link (1963) has derived equations giving the intensity of light in the umbra for light emitted from a point source, accounting for refraction and Rayleigh scattering, as a function of the minimum altitude of the light ray in the earth's atmosphere. For the purpose of investigating the effect of dust extinction on the brightness of an eclipsed moon, Hansen and Matsushima (1966) have calculated the illuminance of the umbral light on the basis of Link's method with necessary modifications to include ozone absorption and scattering by dust and water particles. The brightness distribution over the sun's disk was taken into account in terms of the limb-darkening coefficients. In addition to the extinction of light, the work of Svestka (1948) was applied to take account of the light inserted into the shadow by molecular scattering and the similar processes by dust and water particles. As a result, Hansen and Matsushima concluded that the dust particles may account for large changes in the brightness and colors of the eclipsed moon from eclipse to eclipse. From the observed data of the unusually dark eclipse in December 1963 (Matsushima and Zink 1964), it was found that the additional extinction produced by the increase in atmospheric dust at that time amounted to 0.14 mag. per air mass in the visible region. This compares to the value of about 0.07 mag. per air mass for visible extinction due to the normal amount of dust in the atmosphere (Allen 1963). The result obtained in the foregoing section indicates that the decrease in brightness of the moon in the December 1964 eclipse was not as large as in the preceding year, but it was still much more than either the amount observed in other years or the decrease predicted in the computation by Hansen and Matsushima for an atmospheric model with a normal content of dust particles. It is therefore interesting to make similar calculations for models with different amounts

of dust particles in order to determine a model which best agrees with the observed light curves shown in Fig. 2.

A point M on the surface of the moon in the earth's shadow is illuminated by light coming from different points over the sun's disk and passing through different paths in the earth's atmosphere, depending on the position of each light emitting point on the sun. Hence, the monochromatic brightness of M expressed in terms of that of the same point outside of the earth's shadow will be given by a surface integral of the following form over the entire disk of the sun:

$$D_\lambda(M) = \oint B_\lambda T_\lambda dS / \oint B_\lambda dS, \quad (1)$$

where dS represents a small element of an area on the sun's disk. B_λ is the intensity of light coming from dS , and T_λ is the transmission coefficient for the ray accounting for all changes in intensity during the passage from dS to M . If the brightness distribution over the sun's disk is radially symmetric, B_λ may be given by a limb-darkening formula as a function of the angular distance of dS from the center of the sun's disk. Each process contributing to the atmospheric extinction may be made explicit by writing

$$T_\lambda = T_\lambda^r T_\lambda^R T_\lambda^O T_\lambda^D T_\lambda^W, \quad (2)$$

where the superscripted T_λ 's represent the contributions to the transmission factor from each of the five dominant types of extinction—refraction, Rayleigh, ozone, dust, and water particle (cloud), respectively.

If a spherical symmetry is assumed in the structure of the earth's atmosphere, a ray of light coming from dS and converging to the point M will follow a path which can be specified by a single parameter h_0 , the minimum height of the path above the surface of the earth. If we denote by E the point opposite to M with respect to the center of the earth and at the distance of the sun, we see that the light coming from any point on a circle with its center at E will have a transmission coefficient, $T_\lambda(h_0)$ which is a function of h_0 only. The surface integral (1) may be written as the following two-dimensional integral with respect to h_0 and an angular parameter s along a portion (within the sun's disk) of the circle with center at E :

$$D_\lambda(M) = \frac{\int_{s_1}^{s_2} \int_{h_0}^{\infty} B_\lambda(h_0, s) T_\lambda(h_0) dh_0 ds}{\oint B_\lambda dS}. \quad (3)$$

For a given model of the earth's atmosphere, all the components of T in Eq. (2) can be computed as functions of h_0 . Exact expressions for the T 's are given in the paper by Hansen and Matsushima (1966). (In the

paper by Hansen and Matsushima different units are used for brightness, whereas magnitude is used throughout this paper.) Since our particular purpose is to examine the effect of the atmospheric dust, T_λ^r , T_λ^R , and T_λ^O were computed for standard distributions of molecules and ozone in the atmosphere.

For convenience of calculation, T_λ^D may be written in terms of the extinction coefficient per air mass (in the vertical direction) due to dust particles, β_λ , as follows;

$$T_\lambda^D(h_0) = \exp[-\beta_\lambda L_\lambda(h_0)], \quad (4)$$

where $L_\lambda(h_0)$ is the ratio of the extinction coefficient for a horizontal path with minimum height h_0 to the extinction coefficient in the zenith direction from $h=0$. In general, we can write

$$L_\lambda(h_0) = \frac{2\pi \int_{h_0}^{\infty} \int_0^{\infty} n(a, h) a^2 Q_\lambda(a) \sec z da dh}{\pi \int_0^{\infty} \int_0^{\infty} n(a, h) a^2 Q_\lambda(a) da dh}, \quad (5)$$

where $n(a, h)$ is the number of particles per unit volume and unit radius interval and $\pi a^2 Q_\lambda(a)$ defines the extinction cross section of a single spherical particle of radius a . The zenith angle z may be related to h through an approximate formula

$$\sec z = [R_0/2(h-h_0)]^{1/2}, \quad (6)$$

in which R_0 is the radius of the earth. If dust particles in each size are homogeneously distributed up to a certain height, H , we have from (5) and (6),

$$L(h_0) = (2/H)[2R_0(H-h_0)]^{1/2}. \quad (7)$$

Thus, in this case, we see that the variations in dust extinction between models with different densities of dust particles can be considered by changing the single parameter β_λ . Hence, if the vertical extinction coefficient due to dust particles is determined from independent observations, we have a means of checking the adopted model of the dust layer as determined from eclipse observations. At the same time, we note that $Q_\lambda(a)$ in Eq. (5), which is a complicated function of λ and a , does not enter in Eq. (7) and $L(h_0)$ becomes independent of λ . Thus, as far as the size distribution of particles is independent of the altitude distribution, the color dependency of T_λ^D appears only through the color dependency of the vertical extinction coefficient β_λ . [In the paper by Hansen and Matsushima (1966), an assumption of neutral extinction for dust particles is stated. For the reason cited here, however, this assumption was not necessary for the computations carried out in that paper, either.]

Although the vertical distribution of dust particles depends on the place and time of observation, a number of recent high-altitude balloon observations indicate

that on the average stratospheric dust extends up to 30 km with a maximum at an altitude between 15 and 20 km (Junge *et al.* 1961; Rosen 1964). Below the maximum density the dust content decreases reaching a minimum between 5 and 10 km, this sink apparently being due to rain washout. However, the umbral light must be affected by cloud conditions along a wide range of the terminator region and since the clouds are located at the approximate altitude of the dust sink, it may not be unreasonable to assume that the combined particle density is uniform up to a certain altitude. For this reason, we have calculated three cases for H in Eq. (7); in particular, $H=25$, 20, and 10 km. The variation of the density in each case is considered by taking various values for β_λ . Specifically, we have taken seven values for β_λ between 0.04 and 0.20 mag. per air mass with equal intervals of 0.03 mag. per air mass.

Finally, we also consider the diffused light, i.e., light reinserted into the region of interest through scattering by the molecules and dust particles, which increases by a small amount the brightness of the umbral light, especially near the central region of the shadow. It may be included as an additive term in Eqs. (1) and (3). The molecular diffusion has been estimated through interpolations of Svestka's (1948) values (16.53 at $\lambda=4500 \text{ \AA}$ and 16.68 mag. at $\lambda=7000 \text{ \AA}$) for appropriate wavelengths. The dust diffusion has been also computed from an equation similar to Svestka's and has been found to be less than 17 mag. for all the models under consideration.

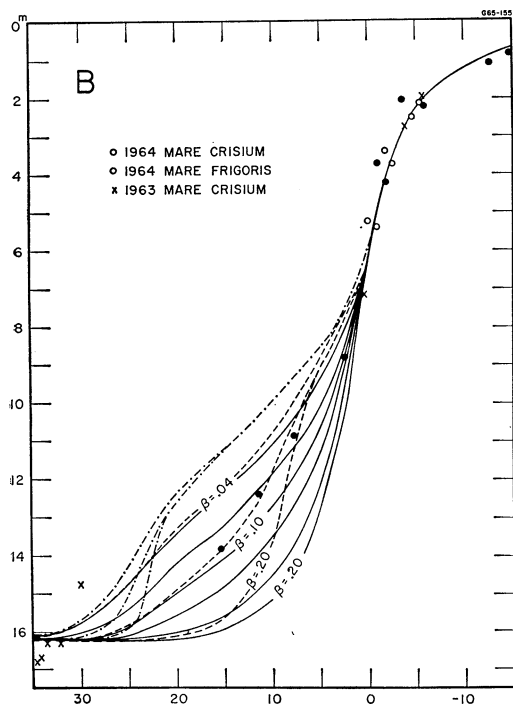


FIG. 4. Comparison of the observed B magnitude and the theoretical calculations for various models, as in Fig. 3.

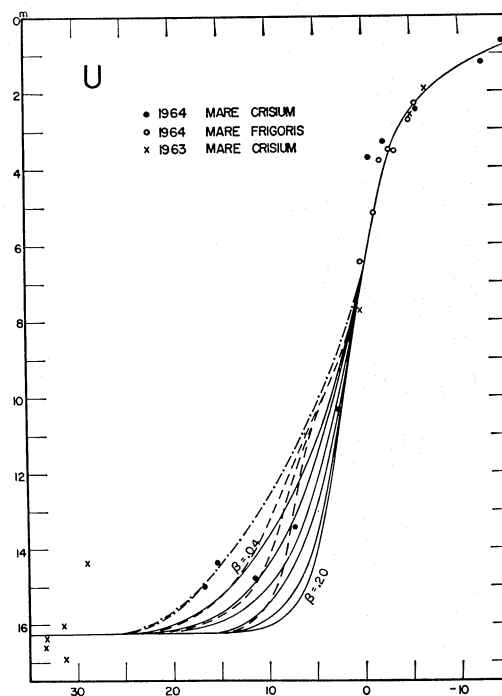
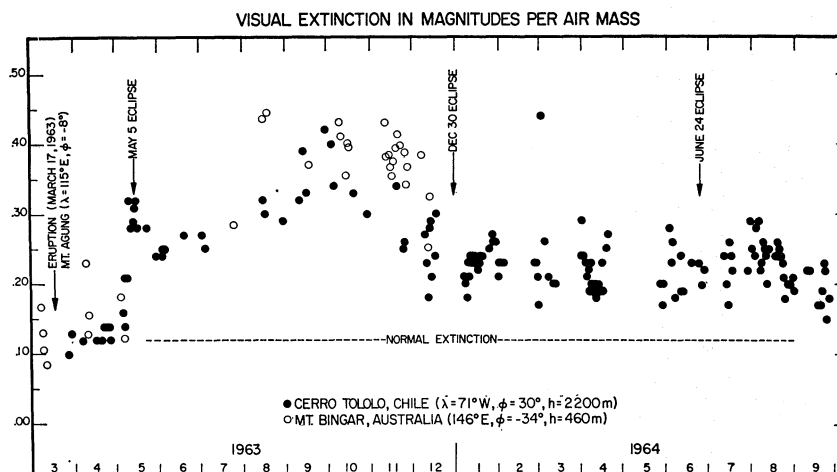


FIG. 5. Comparison of the observed U magnitude and the theoretical calculations for various models, as in Fig. 3.

The results of calculations in each of the V , B , and U colors are compared to observations in Figs. 3, 4, and 5, respectively, in which the observed magnitudes for both 1963 and 1964 eclipses are plotted against the radial distance from the umbra limb r . The solid lines represent theoretical light curves for the case, $H=25$ km with seven different β values as indicated in the figures. The results for $H=20$ and 10 km are distinguished by dashed and combined dashed and dotted lines, respectively. For these two cases, only three curves for $\beta=0.04$, 0.10, and 0.20 are shown in all three figures. We see in Fig. 3 that the best fitting curve for the 1964 eclipse is the case where $\beta_V=0.10$ and $H=25$ km, whereas for the 1963 eclipse it seems to be $\beta_V=0.20$. The blue comparison in Figure 4 indicates that the 1964 observation fits either the case, $\beta_B=0.10$ and $H=20$ km or $\beta_B=0.07$ and $H=25$ km. The ultraviolet observation for the 1964 eclipse compared in Fig. 5 appears to favor the model with $\beta_U=0.07$ and $H=25$ km, although the comparison is less certain in this color because of the small differences between different models. For the 1963 eclipse, the selection of the best fitting model becomes difficult for both B and U colors since the measurements are not available near the outer region of the umbra, as discussed in Hansen and Matsushima's paper.

The average value of β_V due to dust extinction under normal sky conditions is estimated to be 0.06 mag. per air mass (Allen 1963). Hence, a comparison with the above result indicates that the increase in extinction due to the additional dust spread in the atmosphere

FIG. 6. Variation of the photometric extinction coefficient in V after the volcanic eruption of Mt. Agung on 17 March 1963.



was about 0.04 mag. per air mass in December 1964 and 0.14 in December 1963. The visual extinction coefficients have been determined from direct photometric observations by Moreno, Senduleak, and Stock (1964 and private communication) at Cerro Tololo, Chile, and by Przybylski (1965) at Mt. Bingar, Australia, for the continuous periods of 18 and 8 months, respectively, after the eruption of Mt. Agung. The data, which were kindly provided for our use by these astronomers, are plotted in Fig. 6. From the figure we estimate the increase in β_V in December 1963 to be about 0.11 mag. per air mass in fairly good agreement with the above result. After that time, the extinction coefficient decreased very slowly and the additional amount in September 1964 was about 0.06 mag. An extrapolation of the plots to December 1964 seems to indicate the value of 0.04 obtained above and is also in good agreement.

V. DISCUSSION

The accuracy of the photoelectric observations was limited largely by two factors. One was the variation in sky transparency and the resulting uncertainty in the extinction corrections. The other was the brightness of the sky near the moon which imposed relatively large background corrections upon the intensities measured within the umbra, especially in U color. We note that for the purpose of the present paper, it is most important to know the profile of the curve showing magnitude as a function of radial distance from the limb of the umbra. The visual curve is most critical in making estimates of the atmospheric dust, so it is most important that the systematic errors which could distort this curve should not be large. Although the poor observing conditions contributed to the large random errors of these observations, there is no indication that any systematic error was present which would be serious enough to destroy the usefulness of the results. The expected systematic errors would arise

from inaccuracy in determining the mean visual extinction coefficient and gradual changes in the extinction coefficient. The latter would probably have been revealed if the extinction data had been sufficient to average visual extinction coefficients hour by hour.

As the change in the air mass of the moon was about 0.7 for the critical period, the inaccuracy adopted could produce an error of 0.07 mag. in the magnitude range covered by the observed area during this period. A gradual change of 0.1 in the extinction coefficient could produce a distortion in the curve amounting to 0.16 mag. allowing for an average air mass of 1.6 during this period. Although the extinction data were not sufficient to completely ensure that discrepancies of this size were not present, it is unlikely that a total distortion of more than 0.2 mag. in any part of the curve could be introduced without being made evident in either the extinction measurements or in observations of the general sky condition and visibility of stars at low altitudes. Though poor, the sky conditions remained fairly constant overall until shortly before the eclipse had ended. At this time, a somewhat heavier haze developed and observations were discontinued. Because the observation of the unclipped Mare Frigoris was not completed, the adjustment of Mare Frigoris magnitudes to those of Mare Crisium was made using only a slight visual albedo correction. The precise color indices of Mare Frigoris could not be determined and for matching the two sets of observations it was assumed that the color indices of both Mare Crisium and Mare Frigoris areas were identical. The two sets are therefore mismatched for the blue and ultraviolet curves to whatever extent the color indices of the two areas actually differ. This error should be small compared to the other errors present and does not affect the critical visual curve.

The theoretical analysis in this paper is based on the standard atmosphere of the earth with an addition of varying quantities of dust and water particles. The illuminance in the earth's shadow is determined

principally by refraction, Rayleigh scattering, ozone absorption, scatterings by dust and water particles, and diffusion by molecules and dust particles. The first two components are solely determined by the distribution of molecules in the atmosphere, which is now a well known quantity. The atmospheric ozone content varies depending on latitude and seasons. Our calculations for the cases of the vertical ozone content of 0.26 and 0.36 cm STP have shown little difference in the final values of magnitudes in all three colors. Especially since the illuminance at a point in the earth's shadow is determined by contributions of light rays coming through a wide range of the terminator region, the difference due to ozone absorption will be negligible compared to the errors in observations. The diffusion by dust and water particles is also a varying quantity, but as was pointed out in the preceding section, it does not exceed the diffusion by molecules. In any case, calculation shows that the additional light due to diffusion is so small that it has an effect only near the central region of the umbra.

The most important approximation in the present analysis may be the assumption of a homogeneous distribution of dust particles in each different size. Experimental results so far indicate that the distribution of stratospheric aerosols shows significant variations depending upon the place and time of observation. The number of such experiments, especially after the eruption of Mt. Agung, is not sufficient to even estimate the average distribution of dust particles at the time of two eclipses considered in this paper. However, as discussed in the preceding section, if we combine both dust and water particles over a wide range of the earth's surface, the assumption of the homogeneous distribution of these particles may not be an unrealistic approximation. Furthermore, it should be noted that this approximation has an advantage in that we do not have to assume detailed knowledge on the extinction cross section of these particles, which is a complicated function of the index of refraction, size, and shape of these particles.

Despite the above simplification on the size and density distributions of dust and water particles, the agreement shown in the resulting extinction coefficients in V between the present analysis and the direct photometric determinations elsewhere seems rather encouraging. And, in any case, the most important conclusions in this paper, that the atmospheric dust has been present in a very considerable but decreasing amount since the spring of 1963, will not be changed.

The results shown in Fig. 3, 4, and 5 suggest that future observations for the similar purpose should be done in visible or longer wavelength regions and accurate measurements must be obtained near the limb of the umbra.

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